

## HOMEWORK 4

Submission deadline: 17th of Oct, 20:00.

### EXERCISE 1

Write both a recursive function and an iterative function that accept a list of integers and return the maximum stored in the list.

### EXERCISE 2

Intuitively, given some (ordered) set  $S = \{0, \dots, n-1\}$ , a permutation of  $S$  is a rearrangement or reordering of the elements of  $S$ . For example, written as lists, there are six permutations of  $S = \{0, 1, 2\}$ , namely:  $[0, 1, 2]$ ,  $[0, 2, 1]$ ,  $[1, 0, 2]$ ,  $[1, 2, 0]$ ,  $[2, 0, 1]$ , and  $[2, 1, 0]$ . Formally, a permutation of  $S$  is defined as a bijection from  $S$  to itself. That is, it is a function from  $S$  to  $S$  for which every element occurs exactly once as an image value. Let  $S_n$  be the set of all permutations of  $S$ . Given two permutations  $\sigma_1$  and  $\sigma_2$  in  $S_n$ , the composition of  $\sigma_1$  and  $\sigma_2$  (written as  $\sigma_1 \circ \sigma_2$ ) is the permutation obtained by sending each  $i$  in  $S$  into  $\sigma_1(\sigma_2(i))$ .

Questions:

- Write a recursive function which, given some integer  $n$ , outputs the list of all the permutations of  $\{0, \dots, n-1\}$ ;
- Write a program which, given some  $n$  and some tuple  $sig$ , answers the question “is the tuple  $sig$  a permutation of  $\{0, \dots, n-1\}$ ?”. (Note: the program should raise a `ValueError` if the tuple contains duplicate members, or if it does not have the required length.)
- Write a program that takes as arguments two tuples  $sig1$  and  $sig2$  and returns the permutation  $sig1 \circ sig2$ .

### BONUS EXERCISE

Fine-tune the recursive definition of the Fibonacci sequence given in class to make it work on  $n=500$ . One could store the Fibonacci numbers already calculated in a data structure (i.e., list, dictionary, tuple), and ask the program to check if the number has already been calculated.