Normative Multi-Agent Systems

Part II - Deontic logic

(preliminary version)

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Layout

- Introduction
- Syntax and semantics of dyadic standard deontic logic
- Meta-theory of dyadic standard deontic logic

Introduction

Deontic logic

General goal

- Design a language for reasoning about norms
  - Greek *déon*, ‘that which is binding, right’

Requirements

- Formal semantics
- Complete axiomatic characterization
  - Consistency proof: prerequisit for implementation

Guideline

- Start with the simplest possible syntax
- Reserve more complex machinery until the exact limits of the more spartan one are clear

In this tutorial: no time, no bearers of obligations, ...

Dyadic Deontic Logic

- Introduced by Hansson in 1969 under the label DSDL (Dyadic Standard Deontic Logic)
- Motivation: contrary-to-duty (CTD) obligations

Full account in Åqvist (2002)

Syntax and Semantics

Language

Syntax of propositional logic

New building blocks

- $\bigcirc(B/A)$ = B is obligatory, given A
- $P(B/A)$ = B is permitted, given A

A and B are propositional letters

Context-dependent approach to norms

- Truth of a norm usually depends on context
- Dyadic: two arguments

For an unconditional norm, use $\top$ for the condition

Semantics

- Possible worlds (i.e., valuations) are noted x, y, etc.
- A binary relation $\succeq$ (read "greater than or equal to") is used to rank all the possible worlds x, y, ... in terms of betterness.
- Truth-conditions
  - $\bigcirc(B/A)$ true at x iff all the best (according to $\succeq$) A-worlds are B-worlds
  - Similarly for $P(B/A)$ (but with $\forall$ replaced by $\exists$).

P dual of $\bigcirc$, i.e., $P(B/A) = \neg \bigcirc (\neg B/A)$

Example

$n_1 : \bigcirc A$
$n_2 : \bigcirc(B/\neg A)$ (¬ :not)

$x_1 \bullet\ A, B$
$x_2 \bullet \neg A, B$
$x_3 \bullet A, \neg B$
$x_4 \bullet \neg A, \neg B$

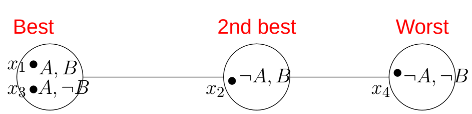
Example

$n_1 : \bigcirc A$
$n_2 : \bigcirc(B/\neg A)$ (¬ :not)

Meaning of $\bigcirc A$

Example

$n_1 : \bigcirc A$
$n_2 : \bigcirc(B/\neg A)$ (¬ :not)



Meaning of $\bigcirc A, \bigcirc(B/\neg A)$

Example

$n_1 : \bigcirc A$
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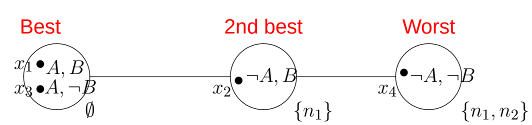


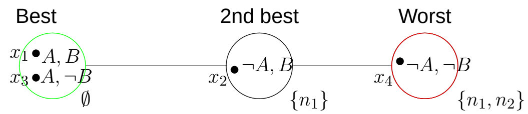
Meaning of $\bigcirc A, \bigcirc(B/\neg A)$

Violation set of $V(x)$ = set of norms that are violated in x

Put $x \succ y$ iff $V(x) \subset V(y)$

Example

$n_1 : \bigcirc A$
$n_2 : \bigcirc(B/\neg A)$ ($\neg$:not)



SDL-ish binary classification of states into good/bad (green/red) ones too crude

Classes of structures

Constraints on $\succeq$

Reflexivity: $x \succeq x$

Transitivity: $x \succeq y$ and $y \succeq z$ implies $x \succeq z$

Totalness: $x \succeq y$ or $y \succeq x$

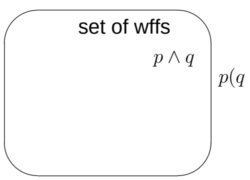
Limit assumption: no infinite sequence of strictly better worlds

Partial pre-order

Total pre-order

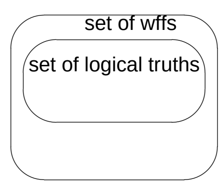
Limit assumption assumed

Meta-theory



❶ Language design
alphabet + formation rules

Output: set of well-formed formulae (wffs) identified

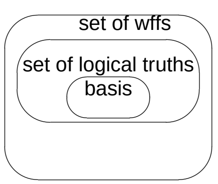


❷ Semantics
Logical truth = truth in virtue of logical form

$$\frac{\bigcirc(B/A) \qquad \bigcirc(C/B)}{\bigcirc(C/A)} \text{ (chaining)}$$

Output: subset of logical truths identified

- Semantic consequence: $\models$
- A is a logical truth if $\emptyset \models A$



❸ Axiomatization
Syntactic consequence: $\vdash$

Success criterium
Completeness theorem: $\Gamma \vdash A$ iff $\Gamma \models A$

Total order case


- Axiomatization problem
  - Weak completeness result ✓
    - Spohn (1975)
    - Åqvist (1987): system **G**
  - Strong or full completeness ✓
    - Parent (2008)
- Consistency ✓
- Decidability ✓
  - Spohn (1975)

Partially ordered case

Partial pre-order 

=

Allowing for conflicts between obligations

$\bigcirc(B/A), \bigcirc(\neg B/A)$ both in

Partially ordered case

Partial pre-order 

=

Allowing for conflicts between obligations

- Axiomatization problem
  - Strong & weak completeness: ✓
    - Goble (2003): system DP

$\Diamond A \rightarrow \neg(\bigcirc(B/A) \wedge \bigcirc(\neg B/A))$ out ($\Diamond$: 'possible')

- Consistency ✓
- Decidability?

Non-transitive case

Call x and y equally good ($x \simeq y$) if $x \succeq y$ and $y \succeq x$.

Argument form

If $\succeq$ transitive, then $\simeq$ transitive
$\simeq$ not transitive
So $\succeq$ not transitive

Non-transitive case

Call x and y equally good ($x \simeq y$) if $x \succeq y$ and $y \succeq x$.

Argument form	*Modus Tollens*
If $\succeq$ transitive, then $\simeq$ transitive	If P, then Q
$\simeq$ not transitive	not-Q
So $\succeq$ not transitive	Therefore, not-P

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Sorites argument

1000 cups of coffes: $C_1, C_2, C_3, ..., C_{999}$

$C_1 \simeq C_2 \simeq C_3 \simeq .... \simeq C_{999}$

Non-transitive case

Call x and y equally good ($x \simeq y$) if $x \succeq y$ and $y \succeq x$.

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$\simeq$ not transitive	not-Q
So $\succeq$ not transitive	Therefore, not-P

Sorites argument

1000 cups of coffes: $C_0, C_2, C_3, ..., C_{999}$

$C_0 \simeq C_2 \simeq C_3 \simeq .... \simeq C_{999}$ but $C_0 \not\simeq C_{999}$

Non-transitive case

Preliminary result: Parent, to appear: Strong completeness result using an alternative language

- Operator: QA “ideally A"
- $\bigcirc(B/A)$ = $\Box(QA \to B)$

Open problems:

- Axiomatize the logic using conditional obligation
- Show decidability
  - On-going work with J. Carmo

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